Skyrmion mass and a new kind of the cyclotron resonance for 2DEG.

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Abstract

The skyrmionic mass was calculated using gradient expansion method. A special cyclotron resonance is predicted with the frequency defined by the exchange energy. The possibility of the extra bound electron is discussed.

The energy and the charge of skyrmion for 2d electron system at high magnetic field were calculated in the papers [1-4] and [5-6]. The difference in the energy expression is connected with the interpretation of the kinetic energy as a constant cyclotron energy in [2-4] and as a differential operator in [5-6]. For the experimental investigation of the existence of the skyrmions it is useful to find specific properties which can be checked by physical measurements. I discuss here the problem of the skyrmion motion as a whole what is not directly connected with it's internal energy. For the calculations I shall use the approach developed in [6,7].

This approach is based upon the transformation of the electron spinors ψ by nonsingular rotation matrix $U(\mathbf{r})$, $\psi = U\chi$ to new spinors χ. One get in such a manner Hartree-Fock equation for spinor χ. It acquires the form

$$i\frac{\partial \chi}{\partial t} = \frac{1}{2m}(-i\nabla_k - A_{0k} + \Omega^l_k\sigma_l)^2\chi - \gamma\sigma_z\chi + \Omega^l_t\sigma_l\chi$$

for the simplest case of the local exchange considered in this paper. Here σ_l are Pauli matrices and $-iU^+\partial_k U = \Omega^l_k\sigma_l$ with

$$\vec{\Omega}^z = \frac{1}{2}(\nabla\alpha + cos\beta\nabla\alpha)$$

$$\vec{\Omega}^x = \frac{1}{2}(sin\beta cos\alpha\nabla\alpha - sin\alpha\nabla\beta)$$

$$\vec{\Omega}^y = \frac{1}{2}(sin\beta sin\alpha\nabla\alpha + cos\alpha\nabla\beta) \tag{1}$$

and γ is the exchange constant. I assume that Euler angles are α, β, α with two equal angles to avoid the singularity in Ω^l for nontrivial degree of mappings Q and suppose $cos\beta = -1$ at the singularity point of $\alpha(\vec{r})$ ([6,7]). This equation for the electrons in the field of rotation matrix U is fully equivalent to the nonrotated Hartree-Fock equation with $\Omega^l = 0$ but the nonuniform exchange term $-\gamma\mathbf{n}(\mathbf{r})\sigma$. Here $\mathbf{n}(\mathbf{r})$ is the unit vector in the direction of the mean spin.

I shall assume that the rotation matrix which adjust the spin direction to a given mean spin direction at any point of 2d plain depends on the position of the skyrmion center $U = U(\vec{r} - \vec{X})$ and calculate the proper term in the skyrmion action due to the time dependence of $\vec{X}(t)$. For the calculation I must find the electron action for the ground state in the field of the matrix U. In the proper electron hamiltonian I shall have the additional perturbation term [6,7]

$$H_1 = iU^+\nabla U\vec{X}_t = -\vec{\Omega}^l\sigma_l\vec{X}_t$$

. In order to find the proper term in the skyrmionic action I must perform perturbation theory calculations in H_1.

Due to isotropy of the system there is no linear term in $\vec{X}_t$ and one must find the second order term in the action $S = iTrlnG$ where G is the electronic Green function. The second order term in the action is

$$\delta S = \frac{i}{2}TrH_1G_0H_1G_0 \tag{2}$$

where G_0 is the unperturbed Green function for Hartree-Fock equation with $\Omega^l = 0$

$$G_0(\vec{r}, \vec{r}', t - t') = \sum_{s,p} \int \frac{d\omega}{2\pi} e^{i\omega(t'-t)} g_s(\omega) \Phi_{s,p}(\vec{r}) \Phi^+_{s,p}(\vec{r}') \tag{3}$$

. Here $\Phi_{s,p}$ are normalized Landau wave functions in Landau gauge and the summation is over all s and p. Matrices $g_s(\omega)$ correspond to the full filling of the lowest spin sublevel for $s = 0$

$$g_0(\omega) = \frac{1+\sigma_z}{2} \frac{1}{\omega - \omega_c/2 + \gamma - i\delta} + \frac{1-\sigma_z}{2} \frac{1}{\omega - \omega_c/2 - \gamma + i\delta} \tag{4}$$

and all other states are empty

$$g_s(\omega) = \frac{1}{\omega - \omega_c(s+1/2) + \gamma\sigma_z + i\delta} \tag{5}$$

where $\delta \rightarrow (+0)$.

The main term with no derivatives of Ω^l and $\vec{X}_t$ in (2) corresponds only to $s = 0$. Also only cross terms are essential with the poles in ω above and under the real axe

$$\delta S = \frac{1}{2} \int Tr(\vec{\Omega}^l \vec{X}_t \sigma_l) g_0(\omega) (\vec{\Omega}^{l'} \vec{X}_t \sigma_{l'}) g_0(\omega) e^{i\delta\omega} \frac{d\omega}{2\pi} \frac{d^2r}{2\pi} dt \tag{6}$$

I perform here the integration over intermediate space coordinates and the summation over p. It is easy to see that only the terms with $l = l' \neq z$ give non zero contribution. Using the isotropy and performing simple integration over ω one gets

$$\delta S = \frac{1}{2\gamma} \sum_{l \neq z} \int \frac{(\Omega^l_x)^2 + (\Omega^l_y)^2}{2} \dot{X}^2 \frac{d^2r}{2\pi} dt = \int \frac{\dot{X}^2}{16\gamma} (\frac{\partial n_i}{\partial r_k})^2 \frac{d^2r}{2\pi} dt \tag{7}$$

Here I use the expressions (1) for Ω^l and introduce the unit vector

$$\vec{n} = (cos\beta, sin\beta cos\alpha, sin\beta sin\alpha)$$

.

It is known that for the state with minimal skyrmion energy for the given degree of mapping Q the value of the space integral [8] is

$$\frac{1}{2} \int (\frac{\partial n_i}{\partial r_k})^2 d^2r = 4\pi |Q|$$

Therefore the kinetic energy term in the Lagrangian is $E_{kin} = \frac{m\dot{X}^2}{2}$ where $m_s = \frac{|Q|}{2\gamma}$ or in usual units

$$m_s = \frac{eB|Q|}{2c\gamma}$$

where B is the external magnetic field. As it was obtained in a number of papers (see e.g [3-5]) the skyrmion has the charge eQ. For the charged skyrmion there are also linear in $\vec{X}_t$ terms in the Lagrangian corresponding to the product of the skyrmion current and the vector-potential of the external magnetic field $\vec{B}$

$$Q \vec{X}_t \vec{A}_0$$

This term can be also calculated by the differentiation of the proper additional phase of the wave function obtained by the translation of the skyrmion charge eQ. Therefore the full Lagrangian for the motion of the skyrmion as a whole is (in usual units)

$$L = \frac{m_s \dot{X}^2}{2} + \frac{e}{c} Q \dot{\vec{X}} \vec{A}_0$$

The hamiltonian conjugate to $\vec{X}$ momentum is $P_i = \frac{\partial L}{\partial X_i}$ which can be considered as a quantum operator with usual commutation relations $[P_i X_i] = i\hbar$. Therefore one have the cyclotron energy for the motion of the skyrmion as a whole

$$\hbar\omega_s = \frac{eB}{m_s c} = 2\gamma$$

. The minimal energy of such motion is

$$\frac{\hbar\omega_s}{2} = \gamma$$

and must be added to the internal energy of skyrmion. In experiments with enough number of charged skyrmions one must observe the cyclotron resonance with the frequency defined by exchange energy per electron

$$\omega_s = \frac{1}{\hbar}2\gamma = \frac{e^2}{\hbar l_B}\sqrt{2\pi}$$

where $l_B = \sqrt{\frac{c\hbar}{eB}}$. The final expression is obtained from the expression for the exchange energy for fully filled Landau level.

The preceding considerations have some important consequence. The thermodynamical enrgy for the system with given chemical potential is the quantum average $< H - \mu N >$ where H is the Hamiltonian, $\mu = (\hbar\omega_c)/2$ is the chemical potential and N is the particle number. The change of the total thermodynamic energy due to the formation of the charged skyrmion is $E_{tot} = E_{int} + \gamma$ where E_{int} is the internal energy not including it's motion as a whole in the external magnetic field. If one put an extra electron in the skyrmion core it's energy will consist from two main parts. One part is the increase in the exchange energy γ because the added electron must have the reversed spin direction according to Pauli principle (all lower states are filled). The other part is the negative Coulomb energy due to the electron interaction with skyrmion charge $\sim (-e^2 Q/L_c)$ where L_c is the skyrmion core size. All other terms in the electron energy are comparatively small ($\sim 1/L_c^2$) and can be neglected for the large size of the core. The added electron make the total skyrmion-electron complex neutral. Therefore there is no correction to it's energy connected with the motion of the complex as a whole in the external magnetic field. The lowest energy of the complex is $E_{compl} = E_{int} + \gamma - const.e^2 Q/L_c$ which is lower than the energy of the charged skyrmion for positive Q and $\mu = (\hbar\omega_c)/2$. One sees that the skyrmion with the large size of the core and positive charge must have the bound electron and become neutral.The spin direction of this electron is reversed to the direction of the average spin in the middle of the core i.e. coincides with the direction of the mean spin at large distances from the core. The results of [5-6] gives the negative thermodynamic energy E_{int} because of the strong reduction of the kinetic energy by $-(\hbar\omega_c)/2$ for $\mu = (\hbar\omega_c)/2$. Therefore such neutral skyrmions must be spontaneously created.

The performed calculations use the assumption of the large size of skyrmion core otherwise the perturbation theory in Ω_l is invalid. The large size of the core requires the small enough g-factor (see e.g.[6]). The conclusions must be numerically checked for the real values of g-factor and magnetically field.

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